Chapter 1

Probablity

1.1 Exponential Families

An exponential family of probability distributions is defined as those distributions whose density have the following general form:

$$p(x|\eta) = h(x)\exp\{\eta^T T(x) - A(\eta)\}$$

Here are the explanations of the parameters in the exponential family formula:

- $p(x|\eta)$: This is the probability density function of the data x given the parameter η. It is a function of the data x and the parameter η.
- $h(x)$: This is called the **base measure**. It depends only on the data x and is often chosen to ensure that the probability density function integrates to 1.
- $T(x)$: This is a vector-valued function of the data x. The vector $T(x)$ is called the **sufficient statistic** because it contains all the information in the data x that is needed to estimate the parameter η.
- η: This is a vector of parameters, often referred to as the **canonical parameter** or **natural parameter**. The distribution of the data x can be fully specified by this parameter.
- $A(\eta)$: This is a real-valued function of the parameter η, known as the **log-partition function** or **cumulant function**. It ensures that the probability density function integrates to 1.

This is a very general form and includes many well-known distributions such as Gaussian, binomial, multinomial, Poisson, gamma, von Mises, and beta distributions

Bibliography